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LETTER TO THE EDITOR

New logarithmic term in the superfluid density scaling in confined geometries

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Abstract. A new logarithmic scaling term is identified in the finite-size form of the helicity modulus, equivalent to the superfluid density fraction, in three dimensions. Implications of the pattern of the scaling behaviour found in the analyses of experimental data are discussed.

The $O(n)$-symmetric order parameter systems have a critical point at non-zero T_c for $d>2$, and they obey hyperscaling for $d<4$. (For $d\geqslant 4$ the critical behaviour is mean-field, with logarithmic corrections at $d=4$.) The quantity associated with the free energy increase due to the order parameter orientational gradients is the helicity modulus $\Upsilon(T)$, see, e.g., Fisher *et al* (1973). In the case of superfluids ($n=2$) in 3D, the helicity modulus is directly measurable. Indeed, $\Upsilon/k_B T$ is proportional to the superfluid density fraction to be denoted ρ. We use ρ in what follows even though most of the discussion applies for general $n=2,3,\ldots$. For $2<d<4$, the critical exponent of Υ can be taken, by hyperscaling, as $(d-2)\nu$, where ν is the correlation length exponent (all these properties are described in detail by Fisher *et al* 1973).

Recent experiments measuring ρ for ^{4}He in confined geometries (Rhee *et al* 1989, Gasparini and Rhee 1990), as well as earlier measurements of the specific heat and superfluid density (Chen and Gasparini 1978, Gasparini *et al* 1984), failed to fit the data by using the standard finite-size scaling forms (Fisher 1971). While the scaling 'data collapse' works quite well, the exponent in the scaling combination $tL^{1/\nu}$ was found to differ from the expected bulk value, $\nu\simeq 0.67$. (Here $t\equiv(T-T_c)/T_c$ so that superfluidity in the bulk corresponds to $t<0$ in our notation.) The effective ν values were found to differ not only from the bulk result but also for different geometries and quantities.

Proposed explanations have ranged from detailed field-theoretical RG calculations (e.g. Huhn and Dohm 1988) to attempts to estimate the effects of the van der Waals interactions (Rhee *et al* 1990). However, no single source of discrepancy has been identified to date to explain all the data. Thus, various possible modifications of the standard finite-size scaling form of Υ and other quantities must be investigated in detail. One such result is reported in this letter. It is *not* claimed that the proposed new scaling term in ρ will suffice to explain the discrepancy in the ν values (as indeed is found in the data analysis by Rhee *et al* (1990); see further in the summary paragraph

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below). However, it is hoped that the proposed scaling formulation will contribute towards the resolution of the experimental versus theoretical discrepancy, the full understanding of which must await future experimental and theoretical work.

For $2 < d < 4$, the finite-size hyperuniversality ideas (Privman and Fisher 1984) can be used to represent the scaling of $\rho(T, L)$ in a finite-size geometry of the characteristic dimension L in the form

$$\rho_{\rm sing}(T, L) \approx L^{2-d} U(atL^{1/\nu}) \tag{1}$$

where the subscript denotes the 'singular part' (as $L \to \infty$), and the scaling function U is universal in that it may depend on the geometry only (including boundary conditions and shape ratios), while all the microscopic interaction details dependence is contained in the metric factor $a > 0$. Note that one can easily extend such scaling forms to allow for other scaling variables besides t, (see, e.g., Privman 1990).

In addition to the part that becomes singular as $L \to \infty$ and contains the leading bulk critical behaviour, for non-periodic boundary conditions there will be 'regular' background terms due to boundary effects such as surface, edge, corner and curvature associated contributions. These background terms were discussed extensively for the bulk free energy (Privman 1988, 1990), and for the interfacial free energy (Privman 1990). Generally, they are inverse powers of L, e.g.,

$$\rho_{\rm reg}(T, L) = \frac{\phi_1(t)}{L} + \frac{\phi_2(t)}{L^2} + \ldots. \tag{2}$$

However, it was noticed by Privman (1988, 1990) that when the 'extensivity', i.e., the leading power of L in (1), of the singular part becomes equal to that of one of the background terms, a 'resonant' logarithmic scaling term may be obtained. In the case of $\rho(T, L)$, this happens in $d = 3$, when the leading power of L in (1) becomes $1/L$.

The precise mechanism of the emergence of the logarithmic contribution here will be similar to that described by Privman (1988) for the free energy density. The amplitude $\phi_1(t)$ in (2), and the scaling function $U(x)$ in (1), where $x \equiv atL^{1/\nu}$, will both develop poles as $d \to 3$. These pole terms have constant residues as functions of t or x, respectively. However, in the limit $d \to 3$ they conspire (see Privman 1988) to yield the new logarithmic scaling term. We only state the final expression

$$\rho_{d=3}(T, L) = L^{-1}\left[\tilde{U}(atL^{1/\nu}) + \omega \ln\left(\frac{L}{l}\right)\right] + \frac{\tilde{\phi}_1(t)}{L} + \frac{\phi_2(t)}{L^2} + \ldots. \tag{3}$$

Here the new scaling function $\tilde{U}$ and the amplitude ω are universal (possibly geometry dependent), while the background terms $\tilde{\phi}_1$, ϕ_2, etc., and the metric factors $a > 0$ and $l > 0$ are non-universal. Note that l is quite arbitrary and it is separated out from the $1/L$ background term only in order to make the argument of the logarithm dimensionless.

Neglecting the $1/L^2$ background, and the non-scaling t-dependence of the $1/L$ background contribution, the leading-order form to fit the experimental data to is

$$L\rho_{d=3} \approx \tilde{U}(atL^{1/\nu}) + \omega \ln\left(\frac{L}{\tilde{l}}\right) \tag{4}$$

where a can be further absorbed into the definition of $\tilde{U}$ (unless one considers several systems and tries to verify the scaling function universality—a situation realisable, e.g. if the pressure dependence of the finite-size superfluid transition is measured). By the 'conventional wisdom' of the critical phenomena theory, the length $\tilde{l} > 0$ is expected to be microscopic.

Note that in the finite-size dominated part of the critical region, i.e. for $L \ll |at|^{-\nu}$, the scaling function can be expanded, $\tilde{U}(x) = u_0 + u_1 x + \ldots$, with the universal coefficients u_0, u_1, etc. In the part of the critical region in which the $T < T_c$ bulk critical behaviour sets in, i.e., for $L \gg |at|^{-\nu}$, while $t < 0$, the limiting form of $\tilde{U}$ must be such that there will be no L-dependence in the purely bulk quantities. A standard argument then yields

$$\tilde{U}(x \to -\infty) = u_\infty |x|^\nu - \nu\omega \ln|x| + O(1). \tag{5}$$

Thus, in the geometries for which the proposed 'surface' logarithmic scaling contribution will be present, we find a universal logarithmic-in-$|t|$ term in the $1/L$ finite-size correction,

$$\rho \simeq u_\infty a^\nu |t|^\nu - \frac{\nu\omega \ln|t| + O(1)}{L} \tag{6}$$

which can be measured if the onset of the bulk behaviour is studied for sizes larger than $|at|^{-\nu}$. Note that u_∞ is universal but it is only a part of the leading bulk amplitude. A similar limiting behaviour applies for $t > 0$, but the amplitude of the leading term is zero (u_∞ is replaced by 0 in (5) and (6)).

It is useful to emphasise that zero-dimensional (fully finite) and one-dimensional (cylindrical) systems have no superfluid transition. The finite-size quantity ρ in (3), (4), (6), etc. is then the apparent value observed in the 3D-like response experiments. On the other hand in the two-dimensional (slab) finite-size geometry ρ is the actual, or the apparent, superfluid density fraction, or the combination of both types of contribution. No clear singularities or jumps were found in the finite-size scaling function in the experimental data fits of Rhee *et al* (1989, 1990). Their results seem to suggest that the 2D superfluid transition in finite-L films lies below the three-dimensional T_c. However, the author is not aware of any theoretical results on the issue of how, if at all, it approaches the 3D transition temperature as $L \to \infty$, which would imply a jump in the finite-size scaling function (see Ambegaokar *et al* 1980, Rhee *et al* 1989).

In the separately published experimental data fit by using (4), by Rhee *et al* (1990), it turns out that $\tilde{l}$ is much larger than typical microscopic lengths. Furthermore the overall data fit is improved only in a limited way by allowing for the additional logarithmic term, *provided* one insists on the bulk ν-value in the scaling term, i.e. a better data collapse is still found if instead of the additional new term one takes ν as an adjustable parameter not necessarily set equal to the bulk correlation exponent value. Finally, there are two observations of a general nature that follow from the present work. First, it is suggested that the analysis of the experimental data may be more complicated than the simple scaling 'data collapse' in terms of the combination $tL^{1/\nu}$. This is in fact true not only for the new scaling term but also for the 'simple' regular background and for higher-order correction-to-scaling contributions not shown in (1), (3) and (4). Secondly, presence of terms which are pole-divergent as $d \to 3$ suggests that the extrapolation of some field-theoretical expansions for finite-size properties may involve previously unexpected features in integer dimensionalities.

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References

Ambegaokar V, Halperin B I, Nelson D R and Siggia E D 1980 *Phys. Rev.* B **21** 1806
Chen T-P and Gasparini F M 1978 *Phys. Rev. Lett.* **40** 331
Fisher M E 1971 *Critical Phenomena, Proc. 1970 E Fermi Int. School of Physics* vol 51, ed M S Green (New York: Academic) p 1
Fisher M E, Barber M N and Jasnow D 1973 *Phys. Rev.* A **8** 1111
Gasparini F M, Agnolet G and Reppy J D 1984 *Phys. Rev.* B **29** 138
Gasparini F M and Rhee I 1990 to be published
Huhn W and Dohm V 1988 *Phys. Rev. Lett.* **61** 1368
Privman V 1988 *Phys. Rev.* B **38** 9261
—— 1990 *Finite Size Scaling and Numerical Simulation of Statistical Systems* ed V Privman (Singapore: World Scientific) ch 1, p 1
Privman V and Fisher M E 1984 *Phys. Rev.* B **30** 322
Rhee I, Bishop D J and Gasparini F M 1990 to be published
Rhee I, Gasparini F M and Bishop D J 1989 *Phys. Rev. Lett.* **63** 410